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representatives from the repayment of the advance. The terms must, of course, be proportionably higher.

The expectation of £1 at the first death of A or B, computed at 6 per cent., is worth ·50280, while the present value of the premium, payable during the joint life, is ·17906, leaving to B a balance of ·32374.

At this rate, for a present advance of £100, there will need to be secured the principal sum of £308·89: but then the lender is bound to pay an annual premium of £6·119 during B's life until his succession to A, and then to hand over to B the policy thus expensively kept up, which policy may, if the succession open late, be of very considerable value.

The difference between the results brought out in the last two cases, shows the propriety of having the destiny of the policy of reassurance clearly laid down in the contract, and of having the computations made according to the very conditions therein contained.

(To be concluded in our next Number.)

On the Determination and Division of Surplus, and on the Modes of returning it to the Contributors.

[Concluded from No. 2.]

HAVING, in our preceding Number, endeavoured to lay down what appears to us to be a correct method of procedure with reference to the determination and division of surplus, we now proceed to consider the modes usually adopted for the return of it to the contributors. Of these we believe the following to be the principal ones practised at the present day:—

1. The share of surplus is returned by way of reduction in the premium next payable.
2. By way of reduction in the t next premiums payable.
3. By way of reduction in all the future premiums.
4. In augmentation of the sum assured.
5. In augmentation of the sum assured, but at a certain rate per cent. per annum thereon.
6. In any one of these ways, but subject to the assured having

previously paid in premiums and interest a sum equal to the amount guaranteed by his policy.

7. In augmentation of the sum assured, but the amount added being subject to reduction, as well as increase, after future valuations.

8. In augmentation of the claims accruing in the interval since the preceding valuation.

When the surplus is returned in reduction of the premium next payable, it is usually in the shape of a per-centage upon the premium; that is to say, each premium is reduced in the proportion which the sum of the premiums annually payable in respect of participating assurances bears to that which is to be divided; so that if we call the former $\int . p$, the latter S, and any given premium p, then the amount allowed in reduction of it will be $\frac{S}{\int . p} \times p$. But we have shown, in our preceding Number, that the share of each member should be $\frac{S}{\int . \phi a^n} \times \phi a^n$ when the interval between the valuations is considerable. When it does not exceed a year, which is almost always the case where the surplus is allotted in the manner under consideration, the operation of interest may perhaps be safely disregarded, and the above expression then becomes $\frac{S}{\int . \phi} \times \phi$; which is identical in value with $\frac{S}{\int . p} \times p$ when ϕ is a per-centage upon p, but not otherwise. Whenever, therefore, the premiums charged are not made up of the true premium increased by a per-centage upon it, the mode of distributing the surplus here alluded to is inequitable, the true one being in accordance with the expression $\frac{S}{\int . \phi} \times \phi$; in other words, the true premium should be omitted altogether from the calculation.

The allotment of the surplus in reduction of a certain number of premiums to be paid thereafter, requires but a simple process. The divisor for each share will of course be the value of a temporary annuity at the given age, for the given term. It will be necessary, however, to have regard to the time of the premium next payable falling due; and hence the divisor will vary from

$1+A^{t-1}$ to A^t. To facilitate the operation, a table may be formed of these values, exhibiting monthly differences, calculated of course at the true rates of mortality and interest.

The same process will have to be followed when all the future premiums are to be reduced, the value of the annuity for the whole term of life being substituted for the temporary one.

The application of the share of surplus in augmentation of the sum assured is attended with no more trouble. Here the divisor will evidently be the single premium for £1, at the true rates, corresponding with the age at the time of the valuation; for where no option is given, it is scarcely necessary to guard against depreciation in lives, which will obviously be of much the same character as those represented in the tables. If an option be given, then it is to be expected that those who are conscious of possessing a vigorous constitution will select the one alternative, whilst those who are in an ailing condition will choose the other. The mean duration of the lives on which the reversions are taken may be slightly diminished by this, and therefore some addition to the real age may fairly be made in such cases. Probably an average one of three years would be more than sufficient for the purpose; for a little consideration will show that the influence which can be thus exerted is very small. As we have said, the addition should be an average one, that is to say, a certain proportion of the expectation in each case, not the same number of years at all ages.

The next method alluded to, viz. the augmentation of the sum assured at a certain rate per cent. per annum, demands more extended consideration. The most remarkable feature in it is, that it entirely disregards the age of the person at whose decease the sum added is to be payable, the only elements taken into account being the amount assured and the duration of the policy. We believe the process is somewhat as follows:—The value of an arbitrary addition, say of £1 per cent. for each year since the date of each assurance is first determined, and a comparison of the sum of these values with the sum to be divided, then affords the means of arriving at the actual addition which this last will suffice for. Hence the relation between the surplus and the total amount added to the assurances may be truly represented; but the proper correspondence between them in each individual case is wholly destroyed. For let us suppose that two persons become members of a society thus dividing, one at 25 years of age and the other at 35; the former will pay, according to the Northampton Table, £2 8*s.* 1*d.* for each £100, and the latter £2 19*s.* 10*d.* Now we believe it will be acknowledged that the Carlisle Table represents pretty nearly the rate of mortality usually prevailing in assurance societies; and we will assume, first, that 3 per cent. is the true rate of interest. After ten years have elapsed, each of the assured receives the same reversionary bonus, the relative *immediate* benefit being in this case as the numbers ·434 and ·509. But the premium *for the risk* in the one case is £1 14*s.* 3*d.*, and in the other £2 4*s.* 8*d.* In ten years, then, with interest, one has overpaid 8·15 and the other 8·94, and these numbers represent the rates at which they should be recouped; but the relation between these is as ·434 to ·476, not ·434 to ·509; and hence, on this hypothesis, the older life gets more than his share.

If we assume 4 per cent. to be the true rate of interest, then the same bonuses allotted to each at the end of ten years will be relatively worth ·345 and ·419, whilst the contribution to the surplus made by each will be as the numbers ·345 and ·377, making matters rather worse than before. And so, let us take what instances we may, we believe it will be found that in none of them does the recipient under this system get the proportion of surplus to which he is properly entitled. It is one, in short, in which the exact adjustment of *individual* rights is entirely sacrificed, for the sake of an arrangement, which, while it is easily intelligible and therefore popular, economises no inconsiderable quantity of labour.

It is proper to mention, however, that there are certain concomitants, in conjunction with which this system presents itself in a less objectionable shape than with others. Thus it seems to adapt itself, although rudely, to the circumstances of an office in which the premiums are made up of the true premium increased by a per-centage; for with such rates, in any given interval, the older the life at its commencement, the more he will have contributed at its termination towards the surplus; whilst the bonus additions for such interval being in each case the same, will also be more valuable in proportion to the increased age of the life; and thus the contribution and the benefit will in some sort correspond. Where the premiums payable are not thus made up, as in the instance of those formed from the Northampton Table, the operation of this system is obviously inequitable. As compared with almost any true table, the additions for extra contingencies in this very commonly adopted rate are either constant, or have a tendency to decrease as the age increases; so that in equal times the contributions to surplus made by the assured are either the same in each case, or diminish as the age of each contributor is greater. But, as we have seen, the value of the reversionary additions is always greater the older the life of the assured, and the additions are thus always at variance with, and sometimes exactly the reverse of, what they ought to be.

We now come to the plan which restricts the allowance of any bonus to such only of the assured as have paid in premiums and interest a sum equal to the amount guaranteed by their policies. In this arrangement, the assured in fact agree to forfeit whatever surplus may have accrued in respect of their assurances, in the event of their dying before a certain period has elapsed; what that period is, is not, so far as we have observed, very clearly defined, for no particular rate of interest at which the premiums are to accumulate is stipulated for. No doubt it will be the same in every case; but that will not make the arrangement either reasonable or equitable. Unless the premiums are no more than sufficient for the risk and actual expenses, in which case there will be no surplus at all to divide, all will pay somewhat too much; and as the lives of some of those who die within the period stipulated for will extend further into it than others, some will forfeit more than others. The intention of the founders of this scheme was no doubt a good one; but in seeking to equalize the advantages to be derived from the ordinary systems of assurance, they have lost sight of the fact, that the true limit of the essential inequalities of any of its systems is the bare value of the risk, taken during small and, for convenience sake, equal intervals of time, as determined by the theories upon which the whole superstructure is raised. In each of the methods we have previously adverted to, it will be observed that clear way is left for a continual approximation to this limit; but in the one under consideration, an artificial barrier is created, which stands in the way of such approximation, and which only ceases to operate when it is too late to remedy the injustice of which it has been the cause..

The next arrangement we have to consider is that in accordance with which the amount added to the sum assured as a bonus may be reduced at subsequent valuations, should a change of circumstances require it. This is practically a harmless provision, since it is extremely unlikely ever to come into operation in a well-conducted society. It will readily be seen, that so long as there is any surplus at all in respect of a given interval terminated, the restriction will be inoperative, inasmuch as the previous additions will not be affected. Supposing them to have been prudently allotted and a sufficient reserve made for expenses, nothing can touch them, not even a decline in the numbers, unless indeed a very inferior class of lives were admitted to make up for such decline. But this is an occurrence to the last degree improbable, since the carrying it out would be against the interest of all the original members. The provision was no doubt introduced as a sort of safety-valve; but its advantages can hardly be considered sufficient to counterbalance the unpopularity with which it must to a certain extent be attended. To the public mind it probably presents a contingency of a formidable character; it is a few only who understand its insignificance.

The last mode we have to record is that by which the surplus of any given year is divided amongst the representatives of those who die in that year. Our readers will probably be as much at a loss as ourselves to discover any good grounds for such an extraordinary provision. It is said to be defended on the principle that copartnership terminates with death, and that the retiring partners are entitled to such profits as exist at the time of their retirement. Why the share of those who remain in the partnership, and who have all more or less contributed to the profits, is not at least reserved, if not distributed, till they in their turn make their exeunt, we do not understand, and we believe it would be quite fruitless to inquire.

Such then are the various methods employed for the purposes in question; as regards the actual determination of the values of the several quantities required, but a very simple series of operations is necessary. Thus, commencing again with the logarithm of the real share of surplus, which it will be remembered is identical with what is called a "cash bonus," or the return to be made when it is to be applied in reduction of the next premium payable, the process will be nearly as follows:—

Log. real share of surplus . . . . £ „ „ „ . . . .
Log. A_x^t . . . .
Log. amount of reduction for t years . £ „ „ „ . . . .
Log. A_x . . . .

Log. amount of reduction in all future premiums . . . . } £ ,, ,, ,, ...

Log. V_x . . . .

Log. amount of reversionary addition . £ ,, ,, ,, ...

The values of A^t_x and A_x must of course be taken out with reference to the time the next premium falls due; and where the option before alluded to is given, the value of V_x must be increased so as to correspond with the expression $V_{x+\frac{e}{n}}$, e being the expectation at age x. In all these cases it is obviously desirable to be prepared beforehand with tables of the quantities brought into play, and of the logarithms of such quantities.

The natural numbers then being found which answer to the several logarithmic ones obtained as above indicated, all that remains to be done is to tabulate them in the order pointed out at a previous page of this article, and the work is complete.

Before we conclude, however, we would say a few words on what we have so frequently adverted to in these pages as the "true premium." It has been objected to the method herein laid down, and urged in favour of the one hitherto most commonly practised, that there is no such thing as a true premium, inasmuch as the elements which constitute it are always varying, are different under different circumstances, and are exceedingly difficult to arrive at even at the best; a little consideration will show that these objections have no weight. It is not pretended that the true premium, or the premium for the risk, can be assigned with perfect accuracy in any given case. What is insisted upon is, that a distinction must always be drawn between it and the premium charged. At the outset, the one is well understood to be, and is sometimes even advisedly made more than the other; and to treat them thereafter as though they were identical, is nothing less than what Bishop Berkeley would call "a shifting of the hypothesis," a proceeding which can scarcely ever fail to lead to erroneous conclusions. Our inability to determine the exact difference between the two may lay us open to errors; but so long as some distinction is maintained, the results we shall arrive at will always be nearer the truth than those obtained from operations in which the distinguishing qualities of the two charges are lost sight of, and the one altogether confounded with the other.